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Fiscal federalism in the provision of merit and impure public goods: who gains, who loses

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Abstract  
In this article we propose a model that explains why fiscal federalism in the real world may be implemented even when efficiency would call for a centralized solution. In a context where Central Government delegates to lower tiers the provision of merit and impure public goods, fiscal federalism in this environment has particular characteristics that have not received due attention: (i) the equalization grant plays a very important role; (ii) cross border provision gives rise to financial agreements that need to be regulated. In a context of full information and in a setting where the traditional benefits arising from fiscal federalism are ignored, our model shows that fiscal federalism, although sub-optimal for the whole community, may be welfare improving for the richest local authority because it reduces the amount of the equalization grant.

Keywords: Fiscal federalism, decentralization, impure public goods, mobility  
Jel Classification: I18, H77
1 Introduction

Policy implementation at national and supranational level must rely on delegated choices in which autonomous decision makers are charged with the responsibility for a specific task.\(^1\) Traditional literature on fiscal federalism\(^2\) suggests that decentralization should follow efficiency principles.

Expenditure decisions should be left to the tier which is better informed on local preferences, while grants might be used for equity and efficiency reasons. Furthermore, fiscal federalism should induce some interjurisdictional competition among political powers resulting from “vote with the feet” (Tiebout 1956) or yardstick competition (Besley and Case 1995). Second generation models (Oates 2005) suggest that the success of fiscal federalism depends on the information the agents possess about either specific parameters (Levaggi and Smith 1994; Levaggi and Levaggi 2011; Akai and Mikami 2006; Snoddon and Wen 2003) or the behaviour of other agents (Petretto 2000), or the effects of their decisions on total welfare (Wildasin 2004; Crivelli and Staal 2008). The first issue has been widely studied in literature and suggests a trade-off between autonomy and control: the local level is better informed than the centre about the relevant parameters that affect welfare and it can strategically use such information. Central Government should then balance the improvement in welfare with the cost deriving from asymmetric information. The last two issues are related since the need for coordination often arises from the presence of spillovers

\(^1\)These tasks may vary from producing a specific good/service to regulating a market.
(Besley and Coate 2003; Ogawa and Wildasin 2009). These problems open a very interesting debate on the distribution of welfare gains deriving from fiscal federalism even in a context where information is symmetric. In our opinion this problem has not received due attention in literature. A second important consideration is that, although sophisticated in its modelling approach, most of the literature models decentralized decision assuming that the good to be produced is a local public good with spillovers. However, most of the services produced at local level are either impure public goods or merit goods. The former are both private goods (increasing utility for the quantity actually bought) and public goods (for the entire amount produced); the latter are private goods whose consumption is financed by the Government for equity/redistribution purposes. These goods can be made available to a specific community either by producing them or by allowing people to receive them outside the local authority boundaries. Fiscal federalism in this context has particular characteristics that have not received due attention: firstly, by its nature, the equalization grant will play a very important role, especially in contexts where income is unevenly distributed; secondly, cross border provision gives rise to financial agreements that need to be regulated.

The aim of our paper is to identify gainers and losers of fiscal federalism in a setting where the traditional benefits arising from

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Most models in this literature study the voting and political process behind decentralization as in Rubinchik-Pessach (2005).


The main examples of goods falling into this category are education and health care.

In some Italian Regions such as Calabria, health care expenditure is financed out of the equalizing grant for as much as 90% of total expenditure.
fiscal federalism (Oates 1972, 1999, 2005) are ignored.

We model the production of a merit/impure public good that produces the same level of utility independently of the government level at which it is produced.\textsuperscript{7} We also explore the welfare properties of three situations: centralization in which the provision of public goods at local level is granted by the central government, fiscal federalism in which the provision is made by an autonomous lower tier and decentralization in which the higher tier makes specific interventions to reduce problems arising from lack of coordination.

The equilibrium conditions and the results are presented for a setting where the quantity of good supplied to the residents coincides with what is locally produced and a more general model where cross border supply is allowed.

In this context where information is complete and symmetric, fiscal federalism without coordination is always sub-optimal for the whole community as one might expect: the lack of coordination means that the quantity of impure public good produced and made available to the community is sub-optimal. However, in our model richer jurisdictions may be better off, because of a reduced tax effort due to a reduction in the equalization grant. This implies that a coordinated solution where local authorities take account of the reciprocal spillovers will never be reached; this is because coordination is not welfare improving for each local authority. To reach a first best solution, the intervention of a supranational authority is necessary which, in the real world, often does not have the necessary information to induce the First Best optimal allocation.

This conclusion has important policy implications because in

\textsuperscript{7}Levaggi and Levaggi (2011) present a more traditional modelling approach where the impure public good produces more utility if supplied at local level.
the real world coordination is necessary for at least two reasons: spillover effects (deriving from the public good aspect of the services produced) and contractual agreement for mobility related service supply (deriving from the merit good aspect of the good produced).

The results of our model add an important interpretation to the present wave of fiscal federalism. Merit and impure public goods for which fiscal federalism is sought seldom present a comparative advantage in being produced locally. The reasons for devolution in this case may be determined by a reduction in solidarity among jurisdictions.

We believe that this aspect related to gainers and losers from fiscal federalism in the provision of merit/impure public goods may also explain the onset of soft budget constraint policies, one of the less desirable effects of fiscal federalism. In fact, wealthy local authorities may become the ultimate gainers from these policies; less wealthy local authorities may try to reduce this power by running into a deficit.

The paper is organized as follows: in Section 2 we present the general framework. In Section 3 the first best in the case of a centralized solution is computed. Sections 4 and 5 show the cases of fiscal federalism and decentralization respectively. A discussion and a numerical simulation is presented in Section 6. Section 7 concludes.

2 The model

The model presented here examines decentralization in a context where the commodity produced is an impure local public good with spillovers. Impure public goods are a fairly heterogeneous category,
varying from impure public goods in their most traditional definition (Musgrave and Musgrave 1989) to spurious merit goods.\textsuperscript{8} In our model, this characteristic of the service is captured by the form of the subsidy. For an impure public good, the usual form is a user charge, i.e. the consumer is asked to pay a fraction of the price of the service produced. When the impure public good is also a merit good, it is usually supplied free of charge, but not necessarily to the entire population.\textsuperscript{9}

A country, whose population is normalized to one, is divided into two local authorities \( i \in \{a, b\} \) of equal size. Each individual has an exogenous money income, \( M^k \) in the range \([M_i, M_i]\), with density function \( \varphi_i(M^k) \). Then, total income in local authority \( i \) is:

\[
Y_i = \frac{1}{2} \int_{M_i}^{M_i} M^k \varphi_i(M^k) dM^k.
\]

Income is used to buy private commodities and one or zero unit of an impure local public good \( H \) whose user charge is equal to \( p_i \). Finally the utility of each individual is accrued by the quantity of \( H \) that is supplied.

The individual’s taste for \( H \) is described by the parameter \( \alpha \) which is assumed to be uniformly distributed in the range \((0, \beta)\),

\textsuperscript{8}The basic difference between a merit good and an impure public good is that the former is in fact a private good that is used to improve income redistribution or to pivot consumers’ preferences towards the use of goods which the planner thinks they should use. We define as spurious merit good a class of services that have this dual characteristic, for example health, education and cultural activities.

\textsuperscript{9}The most representative example in this category is health care which is supplied free of charge or through the payment of a limited fee, but only if the treatment is cost effective.
with density function $\beta^{-1}$. We also assume that $M_i > p_i$, i.e. the poorest individual can afford to buy $H$.\(^{10}\)

The utility function for a representative individual living in local authority $i$ can be written as:

$$V_i^k = M^k - T^k + \max(\alpha^k - p_i) + \phi_i(S_i, S_j),$$

(1)

where $T^k$ is the tax bill paid both at central and local government level in order to finance the provision of merit and impure public goods and other government expenses.

The second term in the expression represents the (private) utility derived from the consumption of $H$. Our framework allows us to model two different cases:

- **traditional impure public goods.** The utility of buying $H$ is given by the difference $\alpha - p_i$ if it is positive, otherwise the consumer does not buy the good and utility is zero. In other words, utility coincides with the demanded quantity.

- **impure merit goods.** In this case $H$ is fully subsidized and its utility coincides with $\alpha$, but the government defines a cut-off in terms of $\alpha$: only the individuals with $\alpha > p_i$ have access to the good itself. In this last case $p_i$ represents the marginal utility of the merit good that the decision maker is willing to finance.

The nature of the impure local public good is captured by $\phi_i(\cdot)$ which allows differentiation of the utility generated by the public good according to where it is produced. As in Besley and Coate\(^{10}\) this assumption will be relaxed in section 3.
(2003), we introduce fiscal federalism by assuming that preferences for the impure public good have the following form:
\[ \phi_i(S_i, S_j) = f_i(S_i) + g_i(S_j), \quad i, j \in \{a, b\}, \]
where \( S_i \) and \( S_j \) are the quantity of good \( H \) produced in the two jurisdictions.

Functions \( f_i(\cdot) \) and \( g_i(\cdot) \) are assumed to be increasing and concave in their argument (decreasing marginal utility at community level), hence the utility of an additional unit depends on where it is produced.\(^{11}\) The level of publicness of the good depends on the functional form for \( f_i(\cdot) \) and \( g_i(\cdot) \). In particular:

1. for \( f_i = g_i \), the good \( H \) is a public good;
2. for \( g_i = 0 \), the good \( H \) is a local public good;
3. for \( 0 < g_i < f_i \), the good \( H \) is a local public good with spillovers.

Total demand in local authority \( i \) is given by:
\[ Q_i = \frac{1}{2} \beta - p_i. \]

Welfare in local authority \( i \) can be defined by the aggregation of equation (1):
\[ W_i = \int_0^\beta \int_{M_k}^{M_i} \left[ (M^k - T^k) + \max(\alpha^k - \theta p_i) \right] \frac{1}{2\beta} \varphi_i(M^k) dM^k d\alpha + \phi_i(S_i, S_j) \]
\[ = \frac{1}{2} \int_{M_k}^{M_i} (M^k - T^k) \varphi_i(M^k) dM^k + \int_0^\beta \max(\alpha^k - \theta p_i) \frac{1}{2\beta} d\alpha + \phi_i(S_i, S_j), \]

\(^{11}\)For a distinction between global public goods and local public goods with spillovers see Levaggi (2010).
which can be written as:

\[
W_i = Y_i - T_i + \frac{1}{4} \beta^2 - 2\theta p_i \beta - p_i^2 + 2\theta p_i^2 \frac{\beta}{\beta} + \phi_i (S_i, S_j),
\]

\[i, j \in \{a, b\}, i \neq j,
\]

where \(T_i\) is the total tax bill borne by jurisdiction \(i\), \(Y_i - T_i\) is income that is available for buying private goods, and

\[
\frac{1}{4} \beta^2 - 2\theta p_i \beta - p_i^2 + 2\theta p_i^2
\]

is total (private) utility from consumption of \(H\). In this case, for \(\theta = 1\), \(H\) is an impure public good, and for \(\theta = 0\), \(H\) is a merit good. Finally \(\phi_i (S_i, S_j)\) is the utility from the public characteristic of \(H\). In this environment, the decision maker has to internalize the externality caused by the consumption of \(H\) via a subsidy that is financed through a linear income tax, partly levied at national level \((t)\) and partly at local level at rate \(\tau_i\).

Here, we assume:

1. \(Y_a > Y_b\), i.e. local authority \(a\) is richer than \(b\);
2. the marginal cost to produce \(H\) is constant and there is no fixed cost.

Given the double nature of private and public good, the beneficiaries of the two characteristics may not coincide. The quantity demanded by residents in each local authority \((Q_i)\) does not necessarily need to coincide with the quantity produced in the same area \((S_i)\). In other
words, we allow for cross border provision of the service modelled.\footnote{Health care, education and cultural goods are good examples of merit goods. The analysis presented here abstracts from transportation cost in order to concentrate on the coordination problem among local authorities.} This implies that to match supply and demand, local jurisdictions have to negotiate a transfer price which will not be equal to marginal cost, given the externality produced.

Fiscal federalism, i.e. complete devolution of the production of good $H$ to a lower tier, causes a welfare loss because the spillover effect may not be correctly evaluated. However, although total welfare is decreasing, fiscal federalism may produce a welfare improvement for some local authorities (the wealthiest ones) because it may reduce the pressure for equalization of resources. This means that a cooperative solution to internalize such externalities may not be feasible and that Central Government’s intervention to correct for spillovers may not be effective if lower tiers have private information or play strategically.

In this paper we concentrate on coordination problems, i.e. we assume that the goods produced at central and local level produce the same utility. In a more general context, the loss deriving from the problems presented in this paper will have to be balanced with the gains outlined by the traditional theory on fiscal federalism.

3 Centralized solution (First Best)

The provision of impure public goods at local level may be granted by Central Government directly or through an agency (centralization), by an autonomous lower tier (fiscal federalism) or by a lower tier with a specific intervention of a higher tier aimed at reducing
the problems arising from lack of coordination (decentralization). The relative advantages of these solutions have long been discussed in literature.\textsuperscript{13} For our setting where there is no relative advantage (in terms of utility) in local provision, the centralized solution corresponds to the First Best.

3.1 General case

Central Government has to find the optimal mix between the national tax rate \((t)\), the local one \((\tau_i)\) and the user charge/number of users \((p_i)\). As in Petretto (2000), the national income tax is used to finance a part of the provision of such good and to redistribute resources. In our model, given the nature of the goods supplied, Central Government wishes to pursue two objectives:

- horizontal equity which implies that a given tax effort should be rewarded with the provision of a uniform amount of public services. This objective can be pursued using an equalization grant \(G_i\);

- affordability which implies that individuals with \(\alpha > p_i\) should not be wealth-constrained. When \(H\) is a merit good, this objective is always attained. For an impure public good, several alternatives are possible. In this analysis we will assume that a subsidy \(I_i = L - M_i\) is paid to all the individuals whose income is below \(L > p\). This means that a quantity

\[
R = \frac{1}{2} \sum_{i=a}^{b} \int_{M_i}^{L} (L - M_k) \varphi_i(M_k) dM_k
\]

\textsuperscript{13}See, for example, Oates (2005), Besley and Coate (2003).
of expenditure has to be financed by Central Government for this purpose;

- the tax rate is written as:

\[ t = \frac{G_a + G_b + R}{Y_a + Y_b}, \]

where \( G_i \) represents the equalization grant which is distributed in a lump-sum form as suggested by Dahlby and Wilson (1994) and Smart (1998):

\[ G_i = \frac{1}{2} \tau^m (\bar{Y} - Y_i), \]

\[ \tau^m = \frac{\tau_a Y_a + \tau_b Y_b}{Y_a + Y_b}, \]

\[ \bar{Y} = \frac{Y_a + Y_b}{2}, \]

where \( \tau^m \) and \( \bar{Y} \) represent the national average surtax rate and the standardized tax base. Both are invariant to each regional fiscal decision, i.e. local authorities do not perceive the effects that their tax rate has on the equalization grant.

When the service is used by residents in one local authority, but produced outside, the jurisdiction producing it is reimbursed at rate \( q \). For this reason, the local tax rate can be written as:

\[ \tau_i = v_i S_i - \theta p_i Q_i + q(Q_i - S_i) - G_i \frac{Y_i}{Y_i}. \]

The problem for Central Government is to find the quantity of \( H \) to be produced, the transfer price and the best output distribution. In actual fact, the maximization cannot be performed for \( q \), the
transfer price. In a centralized system, \( q \) is used as an instrument to redistribute income between the two local authorities; in our model, given the assumption of linear utility as regards disposable income, distribution does not matter. For this reason, the parameter will have to be determined by Central Government using other criteria.\(^{14}\)

The problem for Central Government can be written as:

\[
\max_{p_a, p_b, S_a, S_b} \sum_{i \neq j = a, b} \left( Y_i (1 - t - \tau_i) + \frac{1}{4} \beta^2 - \frac{2\theta p_i \beta - \nu_i^2 + 2\theta \nu_i^2}{\beta} + f_i (S_i) + g_i (S_j) \right)
\]

s.t.

\[
\begin{align*}
\tau_i &= \left( v_i - q \right) S_i - (\theta p_i - q) Q_i - G_i, \\
t &= \frac{G_a + G_b + R}{Y_a + Y_b}, \\
S_a + S_b &= Q_a + Q_b = Q.
\end{align*}
\]

The FOC are derived in Appendix A and can be written as:

\[
\begin{align*}
\frac{\partial f_a (S_a)}{\partial S_a} + \frac{\partial g_b (S_a)}{\partial S_a} + p &= v_a, \\
\frac{\partial f_b (S_b)}{\partial S_b} + \frac{\partial g_a (S_b)}{\partial S_b} + p &= v_b, \\
p &= q + \lambda, \\
\frac{p}{\beta} &= \left( 1 - S^D_a - S^D_b \right).
\end{align*}
\]

The first two conditions can be written as

\[
va - \frac{\partial f_a (S_a)}{\partial S_a} - \frac{\partial g_b (S_a)}{\partial S_a} = vb - \frac{\partial f_b (S_b)}{\partial S_b} - \frac{\partial g_a (S_b)}{\partial S_b},
\]

which can be interpreted in the following way: the allocation of production between the two local authorities should follow an efficiency

\(^{14}\)An alternative may be to use the average cost: \( \frac{va + vb}{\beta} \) or the minimum production cost, \( va \) in our case.
principle by balancing the need to reduce the cost of public provision with the utility both communities derive from the location of the production of that specific good.

The quantity of the impure public good in the two local authorities will be the same in equilibrium. According to the value of $\theta$, $p$ can be interpreted in terms of a price or a demanded quantity. In both cases $p^*$ is chosen to equalize the marginal rate of substitution between income and the impure public good with the price ratio.

The sign of $v_a - v_b$ and the functional forms $f_i(\cdot)$ and $g_i(\cdot)$ determine the direction of the mobility flow. As a special case, for $f_a = f_b$, $g_a = g_b$, then $v_a - v_b > 0$ implies that the flow is from $b$ (the poor jurisdiction) to $a$ (the wealthy one).

Although the effect of $q$ in aggregate cancels out, its value affects $\lambda$, the Lagrange multiplier. This parameter is equal to zero when the transfer price $q$ is chosen to make supply and demand match. In a centralized context where Central Government may choose the level of supply and demand in the two jurisdictions, $q$ may be arbitrarily set. However, if such price does not clear the market, $\lambda \neq 0$, indicating the presence of an equilibrium with rationing.

The further discretion that Central Government has in this case may be used to improve equalization and the level of welfare in the two jurisdictions, but at the cost of severe controls to avoid overspending.
The optimal solution in terms of $p^*, S_a^*, S_b^*, Q_a^*$, and $Q_b^*$ can be substituted in the welfare function:

$$W^* = \frac{1}{2} \frac{\beta^2 - 2\theta p^* \beta - p^{*2} + 2\theta p^{*2}}{\beta}$$

$$+ \sum_{i \neq j = a, b} (Y_i (1 - t - \tau_i^* (q))) + f_i (S_i^*) + g_i (S_j^*),$$

$$\tau_i^* (q) = \frac{S_i^* (v_i - q) - Q_i^* (\theta p^* - q)}{Y_i}$$

$$- \frac{(Y - Y_i)}{(Y_a + Y_b) Y_i} \sum_{i = a, b} (S_i^* (v_i - q) - Q_i^* (\theta p^* - q)).$$

The value of $\tau_i^* (q)$, the optimal local tax rate, depends on the net cost to produce the optimal quantity $S_i^*$ and on the equalization grant. In aggregate they do not have any effect on welfare because they simply determine the allocation of income between the two jurisdictions.

However, the welfare of each single jurisdiction depends on $q$. The lower the $q$ the better-off the less efficient local authority where residents get the impure public good at a very reasonable price. In aggregate, the two effects cancel out and do not affect the equalization grant that is defined on an average tax rate. However, as pointed out before, if $q$ is not chosen to equalize supply and demand, the government will have to implement rationing on the demand for the part of services bought outside each jurisdiction.

### 3.2 Special cases

The model presented in the previous section is a generalization of the framework usually proposed by literature on impure public goods.
where the quantity consumed produces a positive externality. Below we present two special cases of our model in which this assumption is verified.

3.2.1 No mobility

The more traditional case studied by literature on fiscal federalism where the quantity demanded at local level is produced in the same jurisdiction (i.e. cross border supply is not allowed) is a special case of our model. In this case the last constraint in (3) becomes:

\[
S_a = Q_a, \\
S_b = Q_b.
\]

The FOC presented in Appendix A can be written as:

\[
\begin{align*}
\frac{\partial f_a(Q_a)}{\partial Q_a} + \frac{\partial g_a(Q_a)}{\partial Q_a} + p_a &= v_a, \\
\frac{\partial f_b(Q_b)}{\partial Q_b} + \frac{\partial g_b(Q_b)}{\partial Q_b} + p_b &= v_b.
\end{align*}
\]

The interpretation is straightforward: the quantity to be produced in each local authority is equal to the sum of the private marginal utility \((p)\) and the public marginal utility, taking the externalities into due account. It is interesting to note that if the marginal cost in the two local authorities differs, even in the presence of a public good the quantity supplied in the two tiers will be different. This represents an important and interesting difference from the model in which mobility is allowed, a result somehow similar to literature on benefits arising from international trade.
Also in this case, the optimal values $\bar{p}_i^*, \bar{Q}_a^*$ and $\bar{Q}_b^*$ can be substituted in the welfare function:

$$\tilde{W}_i^* = \sum_{i=a,b} \left( Y_i (1 - t - \hat{\tau}_i^*) + \frac{1}{4} \frac{\beta^2 - 2\theta \hat{p}_i^* \beta - \hat{p}_i^* + 2\theta \hat{p}_i^2}{\beta} \right) + \sum_{i \neq j=a,b} f_i \left( \bar{Q}_i^* \right) + g_i \left( \bar{Q}_j^* \right),$$

$$\hat{\tau}_i^* = \frac{\bar{Q}_i^* (v_i - \theta \bar{p}_i^*)}{Y_i} - \frac{(Y - Y_i)}{(Y_a + Y_b) Y_i} \sum_{i=a,b} \bar{Q}_i^* (v_i - \theta \bar{p}_i^*).$$

Welfare is certainly lower than in the previous example, unless mobility is zero in equilibrium in (5). As for welfare distribution, Central Government may simply use the equalization grant, i.e. the margins for redistribution are more limited.

3.2.2 “Global” public goods

Another interesting case that may be considered is represented by the assumption that the public good is produced through demand, not supply. In other words, the welfare function should be written as

$$W = \sum_{i=a,b} \left( Y_i (1 - t - \tau_i) + \frac{1}{4} \frac{\beta^2 - 2\theta p_i \beta - p_i^2 + 2\theta p_i^2}{\beta} \right) + f_a \left( Q_a \right) + g_a \left( Q_b \right) + f_b \left( Q_a \right) + g_b \left( Q_b \right).$$

In this case, the good should be produced in the most efficient local authority. Although the functional form is rather different, the
coordination problems arising in this context are those studied by the literature on global public goods.\textsuperscript{15}

4 Fiscal federalism

In this framework, each local authority sets its own level of taxation and service production according to its preferences and resources. It takes $t$ and $G$ as given, and perceives its budget constraint as hard. Central Government’s role is merely confined to equalizing resources through the lump-sum grant; this actor is the last one to move, i.e. it sets the grant after local authorities have set their own level of expenditure and taxation. Local authorities have the maximum degree of autonomy and we denote it by fiscal federalism.

The first best solution outlined in Section 3 may not be the outcome of a process of fiscal federalism, even in a setting where there is symmetric information between Central Government and local authorities. This usually happens because the local authority does not fully take into account the consequences of its actions on welfare (Petretto 2000). This behaviour usually leads to a suboptimal solution; in our model this is true only for total welfare. Below, in fact, we show that the richer jurisdiction is better-off in the sub-optimal solution.

\textsuperscript{15}See Levaggi (2010) for a review of these issues.
4.1 Cross Border supply

The problem faced by each local authority can be written as

\[
\max_{p_i, S_i} \bar{W} = Y_i (1 - t - \tau_i) + \frac{1}{4} \beta^2 - 2q_p \beta - p^2 + 2q^2 + f_i(S_i) + g_i(S_j)
\]

s.t.

\[
\tau_i = \frac{(v_i - q)S_i - (0p_i - q)Q_i - G_i}{Y_i}
\]  

(8)

The FOC for the problem are derived in Appendix B and can be written as

\[
p_i = q, -v_i + q + \frac{\partial f_i(S_i)}{\partial S_i} = 0.
\]  

(9)

Each local authority does not take into account the spillover effect that its production creates on the neighbour jurisdiction. Furthermore, in their maximization process they take \( q \) as a given parameter, but in equilibrium only a value will clear the market. To reconcile decentralization with market clearing conditions, it is necessary to find the \( q \) that satisfies the optimal conditions (9) and the market clearing constraint. The problem can be solved using a Nash game:

\[
S_i = f_i^{-1}(v_i - q),
\]

(10)

\[
S_a + S_b = Q_a + Q_b = 1 - \frac{q}{\beta}.
\]

After finding \( \bar{q} \) that clears the market, it will be possible to obtain \( \bar{p} \), \( \bar{S}_a \) and \( \bar{S}_b \).

The total quantity produced is lower than in first best because the local authorities do not take the positive externality into ac-
Total welfare can be written as

\[ W = \frac{1}{2} \beta^2 - 2\theta \bar{p} \beta - \bar{p}^2 + 2\theta \bar{p}^2 + \sum_{i=a,b} Y_i (1 - t - \tau_i) \]  

(11)

\[ + \sum_{i \neq j=a,b} \left( f_i \left( S_i \right) + g_i \left( S_j \right) \right), \]

\[ \tau_i = \frac{S_i (v_i - \bar{q}) - Q_i (\theta \bar{p} - \bar{q})}{Y_i} \]

\[- \frac{(Y - Y_i)}{(Y_a + Y_b) Y_i} \sum_{i=a,b} \left( S_i (v_i - \bar{q}) - Q_i (\theta \bar{p} - \bar{q}) \right). \]

Total welfare will be lower than in the first best equilibrium, but this does not necessarily mean that both local authorities are worse off. To explain this, let’s compare (5) with (11). The total quantity of service supplied and consumed is lower than in the first best equilibrium as well as the average local tax rate \( \tau_i \). In general the first effect (the reduction in welfare due to a suboptimal provision of impure public good) should offset the second (increase in welfare due to reduction in the tax rate) as shown by traditional literature (Tresch 2002; Oates 1972). However, in our model the revenue from taxation is also used to finance the equalization grants and cross border shopping. A reduction in the total quantity of impure public good made available to the whole community implies that less resources are needed to finance this component of local taxation. For \( a \), the wealthier local authority, this income effect may offset the initial loss due to underproduction. This gain is directly related to the income difference: the greater the difference between the two local authorities, the better-off the wealthier local authority in a fiscal federalism solution without coordination.

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This result does not depend on the cost of production in the two jurisdictions. As in the centralized model the sign of $v_a - v_b$ along with the shape of the functional forms for the public goods determine the direction of the mobility flow; the welfare gain is simply determined by the distribution of income.

The final element that determines $\tau_i$, the local tax rate, is the price $q$ set for cross border supply. In section 3 we showed that $q$ may be used as a tool to redistribute resources, provided that Central Government is prepared to impose rationing, whereas in fiscal federalism, $q$ is set by the market. The effect of $q$ on the distribution of welfare may be ambiguous, but there are two most likely scenarios: a) $q$ is not used to redistribute income: in this case the sign of the welfare gain is simply determined by reduction in the equalization grant; b) $q$ is used to redistribute resources from $a$ to $b$; if this is the case, the effect simply reinforces the gain deriving from the change in the equalization grant.

4.2 No cross border supply

As for the centralized solution, we can examine the optimal conditions for the case where mobility is not allowed. In this case it is possible to find a solution without a coordination effort between the two local authorities. In fact, each of them maximizes its own utility function and assumes that the quantity produced by the other local authority is set. The FOC for the problem can be written as:

$$\frac{\partial f_a(Q_a)}{\partial Q_a} + p_a = v_a,$$

$$\frac{\partial f_b(Q_b)}{\partial Q_b} + p_b = v_b.$$

(12)

The quantity produced and demanded will clearly be lower than
in the First Best equilibrium as in the case with mobility. Total welfare in this case can be written as:

\[
\tilde{W} = \frac{1}{2} \frac{\beta^2 - \tilde{p}_i^2 - 2\theta \tilde{p}_i \beta + 2\theta \tilde{p}_i^2}{\beta} + \sum_{i=a,b} Y_i (1 - t - \tilde{\tau}_i) + \sum_{i \neq j=a,b} f_i (\tilde{Q}_i) + g_i (\tilde{Q}_j),
\]

\[
\tilde{\tau}_i = \frac{\tilde{Q}_i (v_i - \theta \tilde{p}_i)}{Y_i} - \frac{(Y - Y_i)}{(Y_a + Y_b) Y_i} \sum_{i=a,b} \tilde{Q}_i (v_i - \theta \tilde{p}_i).
\]

Also in this case total welfare is lower than in first best, but this does not necessarily mean that both local authorities are worse off. In this case, the wealthiest local authority is certainly paying less in terms of equalization grant and this effect may offset the loss in utility caused by reduction in the quantity of impure public goods produced.

To explain how this happens, let’s examine Figure 1 where the different effects are depicted.

The first best optimal allocation is represented by the combination \((\hat{Q}_a^*, \hat{Q}_b^*)\). Given a specific level of expenditure, Central Government sets the lump sum grant \(G_i\) so that the net income of each local authority is \((\hat{Y}_a^*, \hat{Y}_b^*)\) and total welfare is \(\hat{W}_a^* + \hat{W}_b^*\). In the fiscal federalism case, each local authority does not perceive the positive externality its production creates and the optimal quantity of impure public good is reduced to \((\tilde{Q}_a, \tilde{Q}_b)\). Expenditure and the average tax rate decreases, there is less need for the equalization grant. This is the reason why the budget constraint shifts from \(aa\) to \(aat\) and from \(bb\) to \(bhb\) respectively. \(B\) certainly suffers a wel-
Figure 1: Centralization vs fiscal federalism: welfare analysis
fare loss, but A may be better off: the quantity of impure public
good produced is lower than the optimal amount, but the income
effect brought about by the reduction in the equalization grant may
compensate such reduction. We can then conclude that while

\[ \hat{W}_a^* + \hat{W}_b^* > \tilde{W}_a + \tilde{W}_b, \]
\[ \hat{W}_b^* > \tilde{W}_b, \]

for A we have

\[ \hat{W}_a^* \geq \tilde{W}_a. \]

The fiscal federalist solution in this case is welfare improving for
the richest local authority which enjoys an income effect deriving
from the reduced burden of the equalization grant.

5 Welfare improving strategies

Traditional public finance literature shows that a coordinated solu-
tion where the reciprocal spillovers are taken into account and paid
for is not reached because of free riding problems. In our model a
coordinated solution that allows a first best optimal allocation to be
reached cannot be achieved because it would mean a redistribution
of resources from local authority A to B. Hence, although feasible
from a theoretical point of view, a fully coordinated solution will
never be the outcome of our game. If Central Government wishes
to improve total welfare, a form of reduced autonomy has to be
introduced.
5.1 Decentralization

Central Government may follow different strategies in a federal context. Its primary objective is to find an optimal trade-off between autonomy and control. For this reason, it may leave local authorities free to set their expenditure and taxation strategies or it may try to induce them to choose a welfare improving equilibrium. Central Government may use several instruments to achieve this objective. In our analysis we will use a matching grant since it is the instrument suggested by literature to correct for spillover. Given the assumption of perfect information, Central Government can find the optimal level of the matching grant by finding the subsidy that allows the externality to be internalized. Tresch (2002) suggests using a unit subsidy equal to the marginal rate of substitution between the public good produced in local authority i and income in local authority j, i.e. the spillover created by each local authority:

In our case (see Appendix C), the optimal rate of the matching grant will be equal to:

$$ r_i^* = \frac{\partial g_j(S_i)}{\partial S_i} \frac{1}{v_i}, $$

and the FOC for each single local authority becomes:

$$ p_i = q, $$

$$ -v_i (1 - r_i^*) + q + \frac{\partial f_i(S_i)}{\partial S_i} = 0. $$

The externality created by the supply of the impure public good is internalized through a matching grant to the jurisdiction that produces the good. However, given that for an impure public good only the quantity actually sold produces benefits to the community,
supply has to match demand. In this model, given the assumption of mobility between the two regions, the marginal price for demand is $q$. As in the previous section $q$ will have to be set to clear the market:

$$S_i = f_i^{-1}(v_i (1 - r_i) - q), \quad (14)$$
$$S_a + S_b = 1 - \frac{q}{\beta}.$$  

In this case, given that $q$ is chosen to clear the market, $\lambda$ will be equal to zero.

The cost of the matching grant will be financed through the national tax $t$. For a decentralized system, in fact, welfare will be written as:

$$W^D = \frac{1}{2} \left( \frac{\beta^2 - p^* - 2\theta p^* \beta + 2\theta p^*}{\beta} + \sum_{i=a,b} Y_i (1 - t^d - \tau_i^d) \right) + \sum_{i \neq j=a,b} \left( f_i (S_i^*) + g_i (S_j^*) \right), \quad (15)$$

$$\tau_i^d = S_i^* \left( v_i (1 - r_i^*) - q^d \right) - Q_i^* \left( \theta p^* - q^d \right)$$

$$= \frac{(Y - Y_i)}{(Y_a + Y_b) Y_i} \sum_{i=a,b} \left( S_i^* \left( v_i (1 - r_i^*) - q^d \right) - Q_i^* \left( \theta p^* - q^d \right) \right),$$

$$t^d = \frac{\sum_{i=a}^b v_i r_i^* S_i^*}{Y_a + Y_b}.$$  

In this case, total welfare is certainly equal to first best, since the quantity produced is the same in both cases. The distribution of welfare between the two local authorities depends on the initial value
of \( q \), the transfer price in First Best and the relative importance of the spillover effect the two local authorities produce. Given that the quantity produced in both cases is the same, the difference in welfare may arise from a different distribution of the fiscal burden. In other words, for each local authority the welfare gain (loss) from first best to decentralization depends on the sign of the following expression:

\[
\tau_i^* - \tau_i^d - \theta^d.
\]  

(16)

If it is positive, the local authority is the net gainer in the decentralization process, if it is negative it will lose. To show how spillovers and transfer price interact, we can observe that (16) can be written as:

\[
\tau_i^* - \tau_i^d - \theta^d = \frac{1}{Y_i} \left( S_i^* v_i - (q^d - q) (S_i^* - Q_i^*) - Q_i \theta p^* \right)
+ \frac{S_i^* v_i r_i^* - S_j^* v_j r_j^*}{2Y_i}.
\]

The first part of the expression depends on the value of \( q \). In particular if Central Government chooses in equilibrium a transfer price to clear the market \( (q^d) \), the two expressions are equal. The second part depends on the spillover effect. In Appendix D, we show that if the utility function derived from the public characteristic is logarithmic as in Besley and Coate (2003), the last part is zero if the spillover effect is reciprocal.

Although this solution represents a welfare improvement for the whole community, it does not imply that both local authorities are better-off. For the same reasons we have presented in section 4.1,

\[\text{See appendix three.}\]
the wealthier local authority will in general be better-off with fiscal federalism rather than with decentralization because the increase in the production of $H$ caused by the matching grant will imply a higher equalization grant.

5.2 No cross border supply

When the quantity demanded in one jurisdiction must exactly match local supply, Central Government has to find a grant that internalizes the positive externality. Using the procedure described in Appendix B, the matching grant can be written as:

$$\hat{r}_i = \frac{2\beta g_j (Q_i)}{v_i \partial Q_i}.$$ 

In this case demand has to be incentivated in order to increase production and to internalize the externality. This is the basic reason why the matching grant is written in terms of marginal rate of substitution from the consumer’s point of view.

Total welfare in this case can be written as:

$$\hat{W}_a = \frac{1}{2} \beta \frac{p_i^* - \theta p_i^*}{\beta} + \sum_{i=a,b} Y_i (1 - \hat{r}_d - \hat{r}_i)$$

$$+ \sum_{i\neq j=a,b} f_i \left( \hat{Q}_i^* \right) + g_i \left( \hat{Q}_j^* \right),$$

$$\hat{r}_i = \frac{\hat{Q}_i^* (v_i (1 - \hat{r}_i) - \theta p_i^*)}{Y_i} - \frac{(Y - Y_i) \sum_{i=a}^b \hat{Q}_i^* (v_i (1 - \hat{r}_i) - \theta p_i^*)}{(Y_a + Y_b) Y_i},$$

$$\hat{r}_d = \frac{\sum_{i=a}^b v_i \hat{r}_i \hat{Q}_i^*}{Y_a + Y_b}.$$
Total welfare is equal to First Best, but the distribution depends on the relative size of the spillover effect the two local authorities produce as shown in Appendix D.

6 Discussion and numerical example

The model presented in this paper shows that even in a context where fiscal federalism is second best because of a lack of coordination in their spending decisions, some local authorities may be better-off in this institutional setting. This is due to the sum of two countervailing effects: a decrease in welfare brought about by the lack of coordination which means that the total quantity of public good is not optimal, and a reduction in the equalization grant. If there is a gainer, it will always be represented by the wealthier regions, while poor regions will certainly be worse-off. In this environment, some forms of decentralization may be more efficient, although they require an amount of information that in the real world CG may not possess. This model clearly shows that solidarity and fiscal federalism cannot be achieved unless the welfare function of each local authority is defined in terms of altruism. In this light we believe that some expressions like “solidal fiscal federalism” that are often used in the political arena should be better outlined. If the actors behave according to a neoclassical utility function, solidarity and coordination will not be the outcome of the game.

We model a class of public goods which we believe may more closely represent the actual supply by local authorities since we concentrate on impure public and merit goods. We show that in this environment the coordination problems characterizing fiscal federalism are important and that it is usually not possible to reach a
first best allocation without the intervention of a supranational authority. Cooperative solutions would be welfare improving for the whole community, but they may not be implemented because they may not be beneficial for all the actors. This result is caused by the equalization of resources that usually characterizes federal systems. The reduction of expenditure due to miss-perception of the spillovers reduces the grant that wealthier regions have to pay to poorer ones. This effect may well offset the initial welfare loss due to under-provision of the impure public good.

In the presence of an impure public good, mobility increases welfare, even if some coordination problems may arise. With decentralization CG is able to force the economy to reach the same level of welfare as in the first best, even if it loses a redistributive tool. In the centralized solution CG may use both the equalization grant and the transfer price \( q \) to redistribute income. In the decentralized solution it is reasonable to assume that \( q \) is set by the market. From a purely theoretical point of view Central Government could still use \( q \) as a redistribution tool and in fact any welfare distribution could be achieved through a suitable choice of \( q \). However, unless \( q \) is chosen to clear the market, the solution will also imply rationing and Central Government will have to impose such rationing and check that each local authority complies with it. In the decentralized solution, in fact, it is the local authority that decides both the production level and mobility. Central Government already exercises indirect control through the grant, but if it also imposes quantity constraints, there is no room for discretion of the local authority.

We now present a numerical example to highlight the results of our paper. As in Besley and Coate (2003) we assume a linear/log
utility for each local authority:

\[
W_i = Y_i (1 - t - \tau_i) + \frac{1}{4} \frac{\beta^2 - 2\theta p_i \beta - p_i^2 + 2\theta p_i^2}{\beta} + w_i \ln S_i + (1 - w_i) \ln S_j,
\]

and we assume that the good produced is a merit good (\(\theta = 0\)).

We have evaluated the different solutions for the following initial parameters

\(Y_a = 16;\) \(Y_b = 6;\) \(w_a = 0.6;\) \(w_b = 0.6;\) \(\overline{S} = 0;\) \(v_a = 5;\) \(v_b = 6;\) \(\theta = 0;\) \(\beta = 20.\)

The results are presented in Table 1.

Welfare reaches its maximum in a system where cross border supply is allowed, as might be expected. In this case production is concentrated in local authority \(A\), but the quantity demanded is the same in both local authorities. The tax rate and the grant depends on \(q\), the price for mobility, as much as the welfare of the two local authorities. The fiscal federalism solution for the most general model is characterized by a relatively high price for cross border supply, which creates an increase in the quantity of good \(H\) produced in \(A\), but a total reduction in demand \((Q_a + Q_b)\). The increase in \(S_a\) and the contextual reduction in \(\tau\) and \(G\) may make local authority \(A\) better-off in this environment, much depends on the price for mobility \(CG\) wishes to fix in a centralized system. The gain in welfare for \(A\) is, on the other hand, unquestionable for the model without mobility. In the latter case, a bargaining solution allowing first best to be reached would be feasible in theory, but \(a\) would never accept it because it implies a lower utility. Finally, through \(CG\) intervention using a matching grant, it is possible to reach first best in this context. In this case, the price for mobility is lower than in fiscal federalism, but the quantity offered is higher.
<table>
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because of the matching grant. It is also interesting to note that for $q = 3.039$ in the first best equations such solution is perfectly replicated in terms of welfare distribution and the local tax rate is equal. It is, however, important to note that this result depends on the type of utility function used and on the assumption of symmetric spillovers as shown in the previous section.

The gain from fiscal federalism for $A$ depends on the size of the equalization grant which in turn depends on the preferences for the merit good\textsuperscript{17} and on the income gap. If the income gap is reduced, the gain from fiscal federalism is reduced and eventually it disappears. In our model this happens for values of $Y_a/Y_b < 1.08$ in the model with cross border shopping and 1.15 for the model where mobility is not allowed.

Finally, it is interesting to note that although total welfare is higher when cross border supply is allowed, jurisdiction $A$ (the wealthier and more productive) would be better-off in a system where mobility is not allowed. The explanation for this result is quite simple: the wealthier jurisdiction, having to pay the equalization grant, is better-off if the production in the poorer jurisdiction is reduced. If cross border supply is not allowed, jurisdiction $B$ sets the quantity of impure public good to be produced according to its marginal cost $v_b$ instead of $q$. Given that $v_b > q$ the production (hence demand) in jurisdiction $B$ is lower. This opens an interesting policy debate about cross border supply and federalism. Unless this policy is imposed by an upper level, it is not likely to be the outcome of coordination between local authorities. This may partially explain the present debate at EU level about patient mobility.

\textsuperscript{17}In our model these preferences are represented by $\alpha$ whose value in turn depends on the interval $[0, \beta]$. 

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across countries (Legido-Quigley et al. 2007; Glinos et al. 2010).

7 Conclusions

In the recent past the process of fiscal federalism has extended the category of goods and services to be provided at local level to include also services that are both impure public goods and merit goods, i.e. they are rival in consumption and can be supplied to local residents also by providers located outside the boundaries of the local authority. In this context coordination among local authorities plays a fundamental role and in this paper we have explored some of the possible reasons why coordination may not be welfare improving.

In a context of full information and spillovers, welfare for the whole community is always suboptimal under fiscal federalism, as may be expected. However, we show that this result does not necessarily imply that each local authority is worse-off. The wealthier and more efficient local authority may prefer fiscal federalism, even if this means a reduction in the total quantity of the impure public good. This is because reduction in the quantity reduces the fiscal burden brought about by the equalization grant. Relevant policy implications arise: fiscal federalism may improve welfare by allowing each local authority to choose its preferred quantity, but it may reduce solidarity among regions. This is a very important problem when fiscal federalism is applied to goods and services such as health care and education which, by their very nature, are often used to redistribute income. The problem is particularly important in countries where income is unevenly distributed across jurisdictions so that the equalization grant plays a very important role in financing expenditure. Our model shows that a positive relationship exists
between the distribution of income and local welfare, i.e. the more income is unevenly distributed, the more the wealthier jurisdiction gains from fiscal federalism. Our model may also explain why in some countries fiscal federalism is often associated with soft budget constraint strategies (Crivelli and Staal 2008; ?): the lack of incentive to policy coordination may lead to such perverse effects. In a more general and traditional context where local authorities have a comparative advantage in producing the merit/impure public good locally, fiscal federalism may still represent the best solution; in this case Central Government will have to find a fine balance between autonomy control and coordination in the decision process (Levaggi and Levaggi 2011). However, it is interesting to keep in mind that devolution and local autonomy may also be advocated to improve local rather than total welfare.

References


Crivelli, E., Staal, K., 2008. Size, spillovers and soft budget con-
 URL http://ideas.repec.org/a/cje/issued/v27y1994i3p657-72.html


URL http://dx.doi.org/10.1007/s10797-005-1619-9


URL http://dx.doi.org/10.1007/PL00011014


A Solution to the Central Government problem

The more general problem can be written as:

$$\max_{p_a, p_b, S_a, S_b} \sum_{i \neq j = a, b} \left( Y_i (1 - t - \tau_i) + \frac{1}{4} \frac{\beta^2 - 2 \gamma p_i \beta - \mu_i^2 + 2 \theta p_i^2}{\beta} + f_i (S_i) + g_i (S_j) \right)$$

s.t.

$$\tau_i = \frac{(v_i - q)(S_i - (\theta p_i - q)Q_i - G_i)}{Y_i},$$
$$t = 0,$$
$$(1 - c) S_a + c S_b = (1 - c) Q_a + c Q_b,$$
$$(1 - d) S_a + d S_b = (1 - d) Q_a + d Q_b.$$

For $c = d = \frac{1}{2}$ mobility is allowed, for $c = 1$ and $d = 0$ mobility is not allowed. The first two constraints can be substituted in the
maximization problem. The Lagrangian is

\[ L = Y_a \left( 1 - \frac{(v_a - q) S_a - (\theta p_a - q) \frac{1}{2\beta} (\beta - p_a) - G_a}{Y_a} \right) \]

\[ + \frac{1}{4} \beta^2 - p_a^2 - 2\theta p_a \beta + 2\theta p_a^2 \alpha \beta \left( 1 - \frac{1}{2\beta} \frac{1}{2\beta} (\beta - p_a) \right) + f_a(S_a) + g_a(S_b) + \]

\[ Y_b \left( 1 - \frac{(v_b - q) S_b - (\theta p_b - q) \frac{1}{2\beta} (\beta - p_b) - G_b}{Y_b} \right) \]

\[ + \frac{1}{4} \beta^2 - p_b^2 - 2\theta p_b \beta + 2\theta p_b^2 \alpha \beta \left( 1 - \frac{1}{2\beta} \frac{1}{2\beta} (\beta - p_b) \right) + f_b(S_b) + g_b(S_a) \]

\[ + \lambda_1 \left( (1 - c) S_a + c S_b - (1 - c) \frac{1}{2\beta} (\beta - p_a) - c \frac{1}{2\beta} (\beta - p_b) \right) \]

\[ + \lambda_2 \left( (1 - d) S_a + d S_b - (1 - d) \frac{1}{2\beta} (\beta - p_a) - d \frac{1}{2\beta} (\beta - p_b) \right), \]

on which the first order conditions are

\[ \frac{\partial L}{\partial p_a} : \frac{1}{2} \frac{q - p_a + \lambda_1 (1 - c) + \lambda_2 (1 - d)}{\beta} = 0, \]

\[ \frac{\partial L}{\partial p_b} : \frac{1}{2} \frac{q - p_b + \lambda_1 c + \lambda_2 d}{\beta} = 0, \]

\[ \frac{\partial L}{\partial S_a} : -v_a + q + \frac{\partial f_a(S_a)}{\partial S_a} + \frac{\partial g_b(S_a)}{\partial S_a} + \lambda_1 (1 - c) + \lambda_2 (1 - d) = 0, \]

\[ \frac{\partial L}{\partial S_b} : -v_b + q + \frac{\partial f_b(S_b)}{\partial S_b} + \frac{\partial g_a(S_b)}{\partial S_b} + \lambda_1 c + \lambda_2 d = 0, \]

which can be rearranged to give the conditions presented in the text.
B Solution to the fiscal federalism problem

In this case, the problem has to be written for each authority:

$$\max_{p_i, S_i} Y_i (1 - t - \tau_i) + \frac{1}{4} \frac{\beta^2 - 2\theta p_i \beta - p_i^2 + 2\theta p_i^2}{\beta} + f_i (S_i) + g_i (S_j)$$

s.t.

$$\tau_i = \frac{(v_i - q) S_i - (\theta p_i - q) Q_i - G_i}{Y_i},$$

$$t = 0,$$

$$S_i = Q_i.$$

The last constraint is relevant only in the problem without mobility. The first two constraints can be substituted in the maximization problem. The Lagrangian is

$$\mathcal{L} = Y_i \left( 1 - \frac{(v_i - q) S_i - (\theta p_i - q) \frac{1}{2\beta} (\beta - p_i) - G_i}{Y_i} \right)$$

$$+ \frac{1}{4} \frac{\beta^2 - p_i^2 - 2\theta p_i \beta + 2\theta p_i^2}{\beta} + f_i (S_i) + g_i (S_i) +$$

$$+ \lambda \left( S_i - \frac{1}{2\beta} (\beta - p_i) \right),$$

on which the first order conditions are

$$\frac{\partial \mathcal{L}}{\partial p_i} : \frac{1}{2} \frac{q - p_a - \lambda}{\beta} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial S_i} : -v_i + q + \frac{\partial f_a (S_a)}{\partial S_a} - \lambda = 0.$$

Mobility allowed In this case, $\lambda = 0$ and the conditions are the same as those presented in the text. Both local authorities will have to agree at a later stage on $q$ in order to clear the market.
Mobility not allowed. In this case $\lambda$ can be substituted back from the first equation into the second one to give the optimal conditions presented in the text.

C Optimal matching grant

In general, the optimal conditions for the supply of an impure public good with spillovers in a community made of $n_i$ individuals can be written as:

$$\sum_{i=1}^{n_i} MRS_{Y_i,H_i} + \sum_{j=1}^{n_j} MRS_{Y_j,H_j} + MRS_{Y_s,H_s} = MRT_{Y,H}.$$

In the fiscal federalism case, each jurisdiction underestimates the marginal rate of substitution because it does not take account of the positive externality and the FOC can be written as:

$$\sum_{i=1}^{n_i} MRS_{Y_i,H_i} + MRS_{Y_s,H_s} = MRT_{Y,H}.$$

To internalize the externality it is sufficient to use a per unit subsidy equal to the aggregate gain of citizens in the other local authority:

$$s_i = \sum_{j=1}^{n_j} MRS_{Y_j,H_j},$$

in the form of a conditional matching grant at rate $r_i = \frac{s_i}{v_i}$.
Mobility In this case, the subsidy is supplied to the producer. From (4) we can write

\[ s_i = \sum_{j=1}^{n_j} MRS_{Y_j', H_j} = \frac{\partial f_j(S_i)}{\partial S_i}. \]

No mobility In this case the subsidy has to be given in terms of demand price reduction, hence the marginal rate of substitution takes account of the marginal cost of taxation. The subsidy in this case will be equal to:

\[ s_i = \sum_{j=1}^{n_j} MRS_{Y_j', H_j} = 2\beta \frac{\partial f_j(Q_i)}{\partial Q_i}. \]

D Welfare comparison

Mobility In order to understand how welfare is distributed, it is necessary to determine the sign of the following expression:

\[ \tau^*_i - \tau^*_i - \tau^*_i, \]

which can be written as:

\[
\begin{align*}
\frac{S_i^*(v_i - q) - Q_i^*(\theta p^* - q)}{Y_i} & - \frac{\bar{Y} - Y_i}{(Y_a + Y_b)Y_i} \sum_{i=a,b} (S_i^*(v_i - q) - Q_i^*(\theta p^* - q)) \\
- \frac{1}{Y_a + Y_b} \sum_{i=a,b} v_i r_i^* S_i^* & - \frac{S_i^*(v_i (1 - r_i^*) - q^d) - Q_i^*(\theta p^* - q^d)}{Y_i} \\
+ \frac{\bar{Y} - Y_i}{(Y_a + Y_b)Y_i} & \sum_{i=a,b} (S_i^*(v_i (1 - r_i^*) - q^d) - Q_i^*(\theta p^* - q^d)).
\end{align*}
\]
Let’s first examine \( \tau_i^* \). In equilibrium, \( S_a^* + S_b^* = Q_a^* + Q_b^* \). It is then possible to write

\[
\tau_i^* = \frac{1}{Y_i} (S_i^* v_i - q (S_i^* - Q_i^*) - Q_i \theta p^*) + \frac{Y_a - Y_b}{2 (Y_a + Y_b) Y_i} \sum_{i=a,b} (S_i^* v_i - Q_i^* \theta p^*) .
\]

The second expression can be written as

\[
\frac{1}{Y_i} \sum_{i=a,b} v_i S_i^* r_i^* + \frac{1}{Y_i} (q d Q_i^* + S_i^* (v_i (1 - r_i^*) - q d)) - \frac{Y - Y_i}{Y_i (Y_a + Y_b)} \sum_{i=a,b} (q d Q_i^* + S_i^* (v_i (1 - r_i^*) - q d)) .
\]

or

\[
\frac{1}{Y_i} (S_i^* v_i - q d (S_i^* - Q_i^*) - Q_i \theta p^*) + \frac{Y_a - Y_b}{2 (Y_a + Y_b) Y_i} \sum_{i=a,b} (S_i^* v_i - Q_i^* \theta p^*) - \frac{S_i^* v_i r_i^* - S_j^* v_j r_j^*}{2 Y_i} ,
\]

and, finally,

\[
\tau_i^* - \tau_i^d - t_i^d = \frac{1}{Y_i} (S_i^* v_i - (q d - q) (S_i^* - Q_i^*) - Q_i \theta p^*) + \frac{S_i^* v_i r_i^* - S_j^* v_j r_j^*}{2 Y_i} .
\]

For a lin-log utility function of the form:

\[
W_i = Y_i (1 - t - \tau_i) + \frac{1}{4} \frac{\beta^2 - 2 \theta p_i \beta - p_i^2 + 2 \theta p_i^2}{\beta}
+ w_i \ln S_i + (1 - w_i) \ln S_j ,
\]

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we have
\[
\tau_i^* - \tau_i^d - t^d = \frac{1}{Y_i} \left( S_i^* v_i - (q^d - q) (S_i^* - Q_i^*) - Q_i \theta p^* \right) \\
+ \frac{(1 - w_i) - (1 - w_j)}{2Y_i}.
\]

If the spillover is reciprocal, the last term is equal to zero and the distribution of welfare between First Best and decentralization depends on \( q \). In particular, if \( q = q^d \) decentralization exactly replicates first best.

No mobility In this case, given that \( Q_i = S_i \) the difference in utility can be written as
\[
\tilde{\tau}_i^* - \tilde{\tau}_i^d - \tilde{t}^d = \frac{\tilde{Q}_i^* v_i r_i^* - \tilde{Q}_j^* v_j r_j^*}{2Y_i},
\]
and for a lin-log utility we get
\[
\tilde{\tau}_i^* - \tilde{\tau}_i^d - \tilde{t}^d = \frac{(1 - w_i) - (1 - w_j)}{2Y_i}.
\]
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