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Adverse Selection in Elderly Care

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Abstract

This paper provides a theoretical model to analyse public funding of family elderly care when two severity type are present (the high and the low), under asymmetry of information and increasing costs. The social planner can redistribute between households, but because of incomplete information he is prevented from observing the type of household. The welfare optimum is characterized both under full and asymmetric information. Under complete information it turns out that the transfer has to be set in such a way to induce equality in the marginal utility of income. The direction of the transfer is no longer clear-cut (both under complete and asymmetric information). Specifically it cannot be ruled out that the transfer flows from the high severity / high cost type to the low severity /low cost type, where intuitively one would expect the opposite

Keywords*: asymmetric information, adverse selection, elderly care, redistribution*
Jel Classification*: D82, I18*

1. Introduction

Social expenditure for the elderly care is doomed to rapidly increase in next years and most industrialized countries will have to cope with this challenge, since population ageing is a widespread phenomenon.

When referring to the care for the elderly, we mean a variety of services, both medical and non medical, intended to meet health and personal needs. In fact, although care may be provided by professional workers, it often happens that caregivers are the relatives of the elderly person (or even volunteers): from this angle we may speak of formal and informal care.

Specifically, the formal care (which incorporates health, nursing and social aspects) is exclusively provided by professional workers. On the opposite, the informal care is usually provided by relatives, volunteer or however non professional workers that provide non medical services aimed at supporting people in their activities of daily living (ADLs). When the informal elderly care is inadequate, then a formal elderly care (skilled care) has to take place, with either a private or a public funding, so that private and public provision coexist.

Even if family informal care has generally to bear monetary costs (mainly depending on private expenditure on goods, factors or even additional formal care), however the main costs it incurs are non monetary. In particular we refer to the cost inherent to the time for care , which is deeply affected by two variables: the level of attention (a sort of quality of care) and the amount (quantity) of care the elderly requires according to his physical and mental condition.

Family informal care cannot be purchased in a market. Nonetheless it is produced (within the family) by a production function whose main input is time. Modelling informal care using usual production functions with decreasing returns , is tantamount to mimic a virtual market in which the shadow price of elderly care is rising with the amount of care provided. A social intervention is required, for instance in the specific form of income transfer outlays. The hindrance is that in many circumstances the individual's severity is a private information, that creates favourable conditions for adverse selection and poses a tricky challenge for the planner intent on social welfare maximization. In fact the individual temptation to lie about the type, in order to result eligible for higher transfers, may avoid a first best outcome to be reached.

In this work we wish to cope with the problem of public funding of family elderly care when two severity type of elderly are present (the high and the low), under asymmetry of information and increasing costs.

We intend to examine the effects of different funding rules drawing on analogies with models from fiscal federalism, where a central government designs grants to support local governmental bodies that face different costs. We consider elderly care as a private good (in the technical sense) whose public interest is captured by the social welfare function. To this extent the paper relates to the literature on private provision of public goods (Cornes and Silva, 2002; Huber and Runkel, 2006). Accordingly we investigate how decentralized Nash equilibrium might approach Pareto efficiency by means of appropriate incentive schemes and under different information scenarios.

In applying the theory of optimal taxation to problem of long-term care the paper presents similarities to the work of Kuhn and Nuscheler (2007), Jousten et al. (2005) and Pestieau and Sato (2008). Similarly to Kuhn and Nuscheler (2007) heterogeneity (and adverse selection) relates to severity rather than to the altruism of the informal care as in Jousten et al. (2005) or to their labour market productivity as in Pestieau and Sato (2008). To some extent the paper mirrors the analysis in Kuhn and Nuscheler (2007) but with some striking and hopefully interesting differences arising from the fact that severity relates to the cost of caring, whereas in Kuhn and Nuscheler (2007) it relates to the benefit of the dependent person. In particular an interesting result we obtain by our model is that the direction of the transfer is no longer clear-cut (both under complete and asymmetric information). Specifically it cannot be ruled out that the transfer flows from the high severity / high cost type to the low severity /low cost type, where intuitively one would expect the opposite (as is indeed the case in Kuhn and Nuscheler 2007).

To conclude, if on the one hand our analysis follows fiscal federalism models, with particular reference to the work of Huber and Runkel (2006) on the other it consistently differs from the current literature on long-term elderly care both in terms of settings and results .

The paper is organized as follows: in Section 2 the model is presented. In Section 3 asymmetry of information is examined, while Section 4 concerns the analysis of categorical block grant with unconditional block grant in a second best scenario. Finally, in Section 5 some concluding remarks are presented.

2. The model

Our analysis follows in most respects the fiscal federalism model provided by Huber and Runkel (2006), which is reinterpreted in the context of long-term care.

We consider households aiming at utility maximization and assume that to each household belongs a person which requires care, provided he has problems with activities of daily living (ADLs). We distinguish between two severity type of elderly: the *high (h)* and the *low (l)*. Whereas to the latter corresponds a low cost of care, to the former is associated a high cost of care. We assume L the number of households characterized by a low cost type and H the number of households with high cost, where *l=1,…,L* and *h=1,…,H; L,H>1.*

The main challenge in modelling informal care depends on the fact that it is provided within the family, and there is no market where it might be purchased. Yet all the choices should be considered as part of an overall maximising problem. To this extent, here it is assumed that each household devotes the time at his disposal both at producing real income, and at caring for the elderly. The consumption of two goods provides utility: the care provided to the elderly (*good x*) and the composite *good y* (in which all the real income is spent). The budget constraint is $R = y + w\ell$, where *R* is the potential income; y, the quantity of private good purchased at the market price normalized to one for simplicity; *w* the wage rate and ℓ the time devoted to the elderly care. Each household produces the quantity of good *(x)* by its own production function $x = \xi(\ell, \vartheta)$; $x_\ell > 0$, $x_{\ell\ell} < 0$; $x_\vartheta < 0$, $x_{\vartheta\vartheta} \geq 0$, whose only input[1] is the time devoted to the elderly care, and the parameter ϑ which is related to the severity type.

Assuming *x* as a monotonic function, its inverse may be written as $\ell = \zeta(x, \vartheta)$; $\ell_x > 0$, $\ell_{xx} > 0$; $\ell_\vartheta > 0$ [2], $\ell_{\vartheta\vartheta} \geq 0$: thus, labelling $E = w\ell$, the budget constraint appears as $R = y + E$. Since we may write $E = x\frac{w\ell}{x}$, labelling $p(x) = \frac{w\ell}{x}$, we can write $E = x\,p(x)$ as well. We know that $\frac{\partial p}{\partial x} > 0$ if $\frac{\partial \ell}{\partial x} > \frac{\ell}{x}$, which is al-

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[1] Other inputs should enter the function for the production of care as those goods and factors privately purchased by households. However, for the sake of simplicity, we assume that the only input which enters the production function is the time devoted to care.

[2] See appendix 1 for details.

ways true if $x=\xi(\ell,\vartheta)=0$ for $\ell=0$, that is to say that *p(x)* is increasing in *x*.

All this boils down to the statement that *(E)* is the virtual expenditure in goods *x* (in care), which might be seen as purchased in a virtual market at a virtual price *p*, the latter rising with the quantity of care provided.

Thus, the cost *E* depends on the quantity of care (*x)* and on the type $i\in\{l,h\}$ of the elder household member. Therefore the virtual expenditure function $E^i(\vartheta_i,x_i)$ for elderly care depends and increases both on the quantity of the care x_i provided, and on the ϑ_i severity parameter, assuming $\vartheta_h > \vartheta_l$. This latter is rendered explicit by the following derivatives: $E^i_x;E^i_\vartheta>0$; $E^i_{xx};E^i_{x\vartheta}\geq 0$ (the subscript indicates the variable with respect to which the *E* cost function has been derived, either at first or second order).

Introducing a government lump-sum transfer τ^i , either zero, positive or negative, the maximization problem that faces the household according to its severity type $i\in\{l,h\}$ is given by $Max\ U^i(y_i,x_i)$, subject to the budget constraint $R^i+\tau^i=y_i+E^i$. We adopt standard assumptions for the U(·) function: it is increasing in *y* and *x* and strictly quasi-concave, as well as that all goods are normal.

In order to maximize the utility function subject to the budget constraint, the household chooses the amount of (*y*)and (*x)* to be provided, according to the following first order conditions (foc)[3]

$U^i_x=U^i_yE^i_x$ or equivalently $-SMS^i_{x,y}=E^i_x$[4]

$R^i+\tau^i=y^i+E^i(\vartheta^i,x^i)$

[3] The subscript indicates the derivative with respect to that variable, i.e., for instance $U_x\equiv\partial U(.)/\partial x$

[4] where $-SMS^i_{x,y}=U^i_x/U^i_y$

Thus, using the implicit function theorem, we can define the optimal values[5] as follows[6]:

$$y^{i*} = Y^i(\vartheta^i, R + \tau^i)\,;\; x^{i*} = X^i(\vartheta^i, R + \tau^i) \qquad (1)$$

Shifting our attention from households' utility to social welfare, we are able to define the first best efficiency conditions from the maximization problem faced by the social planner:

$$\underset{x_i,\,y_i}{Max}\; W = \sum_{i=1}^{L+H} U^i(x^i, y^i) \;\text{ subject to}$$

$$\sum_{i=1}^{L+H}\left[y^i + E^i(\vartheta^i, x^i)\right] = \sum_{i=1}^{L+H} R^i$$

As usual the first order conditions (necessary and sufficient for efficiency, given the concave programming problem) are derived:

$$-SMS^i_{x,y} = E^i_x \quad ;\; i = 1,2,\dots,H+L \qquad (2)$$

$$U^i_y = U^j_y \quad ;\; i = 1,2,\dots,H \;;\; j = 1,2,\dots,L \qquad (3)$$

Conditions (3) require to equalize the marginal utility of good *(y)* among the different severity type of households. Assuming that the means at social planner disposal to get its policy goal consist on a lump sum transfer, equal in amount among all the same type households, then the maximization goal entails a solution for the following problem: $\underset{\tau^i}{Max} \sum_{i=1}^{L+H} U^i(x^{i*}, y^{i*})$ subject to $\sum_{i=1}^{L+H} \tau^i = 0$ where x^{i*}, y^{i*} are the household equilibrium values provided by eq. (1).

Analysing the corresponding focs it emerges that the condition

$$\frac{\partial U^h}{\partial \tau^h} = -\frac{\partial U^l}{\partial \tau^l} \qquad (4)$$

has to be met, since the constraint $\sum_{i=1}^{L+H} \tau^i = 0$ force the transfer τ to be opposite in sign for each type of household. The economic hint underlying this condition is straightforward: the social planner

[5] Which represent as well the demand function along the optimal path

[6] $J^i(x^i, y^i) = E^i_x \Rightarrow \begin{cases} y^{i*} = Y^i(\vartheta^i, R + \tau^i) \\ x^{i*} = X^i(\vartheta^i, R + \tau^i) \end{cases}$

transfers money from one type of household to the other as long as the marginal utility of the receiving household is higher, in absolute value, with respect to the giver's. The optimal point is reached when eq. (4) is satisfied.

It is possible to prove[7] that the optimum transfer outlays $\tau^{h*}=-\tau^{l*}$, according to equation (4), induce a Nash equilibrium characterized, for all the households, by equal marginal utilities for good *(y)*, or $U_y^{h*} = U_y^{l*}$. That is to say that such a Nash equilibrium is a Pareto equilibrium as well, so that the economic meaning of this statement is that social planner for its maximization goal can use lump-sum income transfers, and in so doing it has simply to control income marginal utilities of the households.

Differentiating the utility function of the high severity type by ϑ^h we get

$\frac{\partial U_y^{h*}}{\partial \vartheta^h}[x^{h*}(\vartheta^h, R^h+\tau^h), y^{h*}(\vartheta^h, R^h+\tau^h)] >,< 0$, which in turn implies (by eq. 2)

$$\frac{\partial\left(\frac{U_x^{h*}[\cdot]}{E_x^{h*}[\cdot]}\right)}{\partial \vartheta^h} >,< 0 \qquad \Rightarrow \qquad \frac{\partial U_x^{h*}}{\partial \vartheta^h} E_x^{h*} - \frac{\partial E_x^{h*}}{\partial \vartheta^h} U_x^{h*} >,< 0$$

If we define $\varepsilon = \dfrac{\frac{\partial U_x^{h*}}{\partial \vartheta^h}}{U_x^{h*}} \Bigg/ \dfrac{\frac{\partial E_x^{h*}}{\partial \vartheta^h}}{E_x^{h*}}$ then $\varepsilon >< 1$

Thus, $\varepsilon>1$ if $d\vartheta^h$ let the *per cent* variation of marginal utility be higher than the corresponding variation of *per cent* marginal cost of care (good *x)*. In that case $U_y^{h*} > U_y^{l*}$ occurs and the social welfare is maximized by a transfer $\tau>0$, i.e. the low severity household must finance the high severity one. Vice versa, in the case that $\varepsilon<1$, then the transfer has to move from the high severity to the low se-

[7] The proof is reported in appendix 2

verity one; finally if $\varepsilon=1$, then τ has to be set equal to 0. This result can also be provided in terms of elasticities[8] since the sign of τ depends on the elasticities $\eta_W = \frac{\partial(U_x)}{\partial\vartheta}\frac{\vartheta}{U_x} >,=,< \frac{\partial(E_x)}{\partial\vartheta}\frac{\vartheta}{E_x} = \eta_E$. In fact it is sufficient to multiply both the numerator and the denominator of the equation for ε, by θ, to realize that condition $\varepsilon >,=,< 1$ becomes condition $\eta_W >,=,< \eta_E$.

The direction of the transfer outlays to the households is not defined a priori.

In analogy to Huber and Runkel (2006) we find that transfers should flow from low to high (from high to low) if and only if the elasticity of the marginal utility from care exceeds (falls short of) the cost elasticity of care[9]. Thus, if the marginal caring effort increases much more steeply in severity than the benefit the family obtains from this, then it may be optimal for transfers to flow from the severe type to the less severe type. This is because given that the (marginal) benefits from care do not vary much with severity from a utilitarian perspective, it is then optimal to subsidise production within the lower cost household.

This counterintuitive result can arise under two circumstances (see appendix 3 for a formal exposition of the two cases):

i) Expenditure on care is lower for the more severe type. In intuitive terms this implies that severe cases receive less attention from their families (although the latter take full account in their utility of the benefits from caring). These families, in turn, then enjoy a greater consumption. In this case, there may be some intuition that the government then seeks to tax away some of the extra consumption for these families. This may be regarded as a

[8] Huber, B. and Runkel, M. (2006).

[9] The result that transfers may flow both from low to high severity types but also the other way round is, in our opinion, of some interest when applied to the provision of long-term care, although it is not new per se.

rather "artificial case". In the real world the problem is that informal carers typically suffer a loss in their income (see e.g. Heitmüller and Inglis, 2007) and one would expect this loss to increase in the degree of severity.

ii) Expenditure on care (i.e., the attention level) is higher for more severe types and consumption is lower. Furthermore, despite the greater caring input, the benefit from care is lower. This case, which seems to be very plausible, implies that families work hard for their severely dependent members but cannot really help them. In this case, the counter-intuitive result arises if (and only if) the benefits from care and consumption are substitute goods in the utility function (i.e., if there is a positive cross-derivative). The mechanism behind the result is then the following: as low benefits from care are realised for severely dependent types, this also implies a low marginal utility of consumption for these families. Therefore the utilitarian government redistributes towards the low types[10].

3. Information asymmetry and transfers

In this section the incomplete information case is considered. The social planner is aware of the fact that there are low and high type households, but he is prevented from associating the right type to each one. Information concerning potential income, utility function and cost structure are at his disposal, but it is not the level of care provided to the elderly. However he can observe the expenditure on it (E^i) and the expenditure on y^i, for $i=h,l$.

The social planner aims at welfare maximization by means of lump sum transfer.

[10] This result illustrates the scope for utilitarian welfare functions to generate perverse outcome and probably it does not provides a good basis for policy-making

That is to say that he has to find a value $\tau°$ that maximises the social welfare, when both households may opt for receiving the transfer $\tau°$ conditional on spending E°, or paying the tax ($-\tau°$) and no auditing.

Note that the social planner cannot offer contracts with the Paretian $[\tau^*, E_h^*]$ or $[\tau^*, E_l^*]$ because of cheating: the household's type which has to pay the transfer, could pretend to be the other.

The social planner maximization problem can be described as follows:

$$\underset{\tau}{Max} W[x^i, y^i, x^j, y^j] = U^i[x^i, y^i] + U^j[x^j, y^j]$$

subject to :

- budget constraint

$$y^i + E^i(\vartheta^i, x^i) = R^i + \tau \qquad \text{(associated lm: } \lambda^i\text{)} \qquad (5)$$

$$y^j + E^j(\vartheta^j, x^j) = R^j - \tau \qquad \text{(associated lm: } \lambda^j\text{)} \qquad (6)$$

- incentive compatible constraints[11]

$$U^i[x^i, y^i] \geq U^i\left\{\psi_x^i[E^j(x^j, \vartheta^j), \vartheta^i], y^j\right\}\Big|_{R^i \geq R^j} \quad \text{(associated lm: } \mu^i\text{)} \quad (7)$$

$$U^j[x^j, y^j] \geq U^j\left\{\psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j], y^i\right\}\Big|_{R^j \geq R^i} \quad \text{(associated lm: } \mu^j\text{)} \quad (8)$$

- non negativity constraints

$$x^i, y^i, x^j, y^j \geq 0$$

[11] The incentive compatible constraints are required to avoid the cheating strategy, that is to mimic the other severity type. These constraints simply state that the utility deriving to the *i* type household when it sincerely reveals its type, has to be not lower than the utility deriving to that household from cheating, i.e. when it falsely declares to be of the other type. The goal is to avoid any incentive to lye.

x_j^i is the level of care that the cheating household *j* has to provide in order not to be detected by the social planner. It is possible to show[12] that $x_i^j = \psi_x^j(e^i, \vartheta^j)$ [13].

Considering by hypothesis the case in which τ is positive[14], implies that household *j* is taxed while household *i* is subsidized. This assumption allows us to set $\mu^i = 0$ given that the incentive compatibility constraint of eq.7 is not binding. In fact the receiving household *i* has no advantages to misrepresenting its type declaring to be the other.

Proposition 1: *if $\varepsilon = 1$, second best and first best coincide*
if $\varepsilon < 1$, then a second best is attainable by subsidizing the low type
if $\varepsilon > 1$, then a second best is attainable by subsidizing the high type
In the second best scenario we expect: $U_{x^i}^{i\circ} > U_{x^i}^{i*}$; $U_{y^i}^{i\circ} > U_{y^i}^{i*}$ and $x^{i\circ} < x^{i*}$; $y^{i\circ} < y^{i*}$

In the trivial case of $U_y^h = U_y^l$ (or $\varepsilon = 1$) the first best and the second best allocation coincide and the transfer τ has to be set equal to zero.

But, in general, the initial equilibrium might be characterized either by $U_y^h > U_y^l$ or $U_y^h < U_y^l$, i.e., $\varepsilon > 1$ or $\varepsilon < 1$.

In the first case where $U_y^h > U_y^l$, in order to maximise social welfare, it is necessary to rise U^h, i.e. the social planner has to tax l and return to h the correspondent transfer outlay. The incentive compa-

[12] See appendix 5 for details.
[13] The first order conditions (foc) required for efficiency are reported in appendix 6
[14] See the previous section for details

tible constraint expressed by eq.8 requires that the transfer τ has to be set in such a way to render household l indifferent between pay the tax τ and choose its optimal expenditure (E^l, y^l) or to receive the transfer τ conditional to the expenditure (E^h, y^h), which is the optimal expenditure for the other (receiving) household h. Actually, as we have noted above, this is tantamount to say that for household (l) the positive transfer τ is conditional to the quantities (x_h^l, y^h).

In the opposite case, when it is $U_y^h < U_y^l$, in order to maximise the social welfare it is necessary that U^l rises, i.e., social planner has to tax *(h)* and must give to *(l)* the correspondent transfer outlay.

The incentive compatible constraints have to grant that for *(h)* it is indifferent to pay the tax τ and freely choose the expenditure *(E^h, y^h)* or to receive the transfer τ conditional to the expenditure *(E^l, y^l)*, in other words for household *(h)* the positive transfer τ is conditional to the quantities *(x_l^h, y^l).*

The incentive compatible constraint does not permit to join the condition of equality between the marginal utility (with respect to the composite good) for the two household types (see appendix 6 for details). The final equilibrium outcome will turn out to be a second best outcome. Even if further improvement might be obtained by a different resource allocation, it is avoided because of the incentive compatibility constraint. The efficient equilibrium outcome where $U_y^i = U_y^j$ is not attainable in presence of asymmetry of information because this condition creates favourable condition for *j* to lie about its type and thus to adopt a strategic behaviour.

It is interesting to compare the outcome of the first best scenario with this latter characterized by a lack of information. Looking at a.6 and a.8 we note that the marginal utility for the receiver *i* with respect both to good x and good y, is greater if compared with the complete information scenario, and as a consequence, the quantity for the two goods will be lower. Noting by * (star) the outcome emerging from the complete information case and by ° (circle) the outcome in the incomplete scenario, we can summarize as follows: $U^{i\circ}_{x^i} > U^{i*}_{x^i}$; $U^{i\circ}_{y^i} > U^{i*}_{y^i}$ and $x^{i\circ} < x^{i*}$; $y^{i\circ} < y^{i*}$.

Proposition 2: if $\frac{U^j_{\psi^j}\psi^j_{e^i}}{U^j_{y^i}} \neq 1$ *then the recipient's consumption is distorted and the second best expenditure on care has to be forced*

Lump sum transfer in the second best scenario may avoid the receiving household to meet his efficiency conditions (according to the Nash behaviour) causing distortions in the recipient's spending decision[15]. In particular we may note underconsumption on the level of care (with respect to the efficiency rule) if $\frac{U^j_{\psi^j}\psi^j_{e^i}}{U^j_{y^i}} > 1$ but even overconsumption in the opposite case[16]. This result implies that the social planner has to force the recipient to a lower expenditure on care (or consistently to a lower level of care) with respect to his attitude in order to attain a second best outcome. In fact, if not, the incentive compatibility constraint couldn't be met and misrepresentation of type is likely to emerge. Indeed two conclusion regarding the household which receives the subsidy deserve our at-

[15] This result sensibly differs from that of Huber and Runkel (2006), in fact we allow for the particular case of no distortion in the recipient when the condition $U^j_{\psi^j}\psi^j_{e^i} = U^j_{y^i}$ (see a.6 and a.8) is met. On the other hand if this equality is not verified then a distortion emerges.

[16] Also this result is new with respect to Huber and Runkel (2006).

tention: the first concerns the role of information in bounding its provision of care, the second concerns the subsequent harmful distortion in its behaviour. Therefore it is possible to state that under the condition $\frac{U^j_{\psi^j}\psi^j_{e^i}}{U^j_{y^i}} < 1$ the Nash behaviour of the receiving household would enable for a higher level of care with respect to the social (second best) optimum[17]. The condition $U^j_{\psi^j}\psi^j_{e^i} = U^j_{y^i}$ has a straightforward interpretation: in the limiting case in which the marginal effect on the contributor's utility of the recipient's expenditure on *x* is equal to the contributor's marginal utility with respect to the recipient's expenditure on good *y*, then no room for distortion is left, or evenly, the second best outcome coincides with the outcome coming from individualistic Nash behaviour of households (given the second best transfer).

With reference to the receiving household and with respect to the first best outcome, it is possible to state that underprovision for the goods *x* and *y* is always detectable. That result comes as a direct consequence of the incentive compatible constraint which binds the social planner to a suboptimal amount for the $\tau : \tau^{\circ} < \tau^{*}$.

On the other hand eqs. a.7 and a.9 imply the condition that $U^{j\circ}_{x^j} < U^{j*}_{x^j}$; $U^{j\circ}_{y^j} < U^{j*}_{y^j}$ and hence $x^{j\circ} > x^{j*}$; $y^{j\circ} > y^{j*}$. The contributor provision of good *x* and consumption of good *y* in the asymmetric information context exceeds the first best one.

[17] Using CES utility function as: $\left(x_i^{\frac{\beta-1}{\beta}} + y_i^{\frac{\beta-1}{\beta}}\right)^{\frac{\beta}{\beta-1}}$, $\left(x_j^{\frac{\sigma-1}{\sigma}} + y_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ and assuming for instance:
β=1,8; θ_i=2; σ=2; θ_j=1; $R_i \leq R_h$; it is possible to check what just stated.

Proposition 3: if the income of the receiving household is not lower with respect to the contributing, then a first best policy can be implemented.

In the case that household to be taxed enjoys an exogenous income that falls short of the income of the household to be subsidised, then a first best allocation can be implemented. The reason is that in order to mimic the receiving type, a contributing household would have to attain the same levels of observable expenditure. Naturally, this is impossible if the exogenous income is lower.

The constraint that binds is the budget ($y^j + E_x^j = R^i + \tau$) rather than the incentive compatibility one (eq.5 and eq.6). It happens that the income of the contributing household $(R^i + \tau)$ is not sufficient to match (E^j, y^j). This assertion can be generalized as follows:

if severity type j is the cheating household, i.e., the household that misrepresents its type in order to receive the subsidy, then its budget constraint has to meet the condition:

$$R^j + \tau \geq y^i + E^i \tag{9}$$

where i is the receiving household.

The receiving household meets its budget constraint, in order to maximize its utility, by equality, i.e.,

$$y^i + E^i = R^i + \tau \tag{10}$$

It clearly emerges from eq.9 and eq.10 that the binding budget constraint which allows the donor household to declare to be the other type and meet the individual budget constraint can be simply synthesized by the condition:

$$R^j \geq R^i$$

4. The voucher policy

The previous sections' analysis shows that if asymmetric information is assumed, then a second best is the only possible outcome, but distortion at recipient household is a likely consequence. The recipient household, when left free to decide about the expenditure on care, will opt either for a lower or a higher level with respect to the second best optimum.

As we have already shown, when adopting a lump sum tax, the social planner is confident that he will not cause any distortion at the contributor. The distortionary policy emerges with reference to the receiver. This point is crucial assuming the household autonomy in the spending decision.

Proposition 6: by lump sum transfer and voucher[18] *for care a second best is attainable under the condition* $e^{i^\circ} \leq e^{j^\circ}$

Adopting a policy consisting on *voucher* that allows for a certain level of expenditure on x along with *a lump sum transfer* the social planner is able to implement, under specific conditions, a second best outcome. In general we can state that a second best is a possible outcome but that result cannot be taken as granted[19]. In our model where households are characterized by different income, utility and cost function, it may happen that the second best optimal expenditure of the subsidized household is lower, greater and even equal to the second best optimal expenditure of the taxed household. In other words, defining the second best optimum by the index :

$E^{i^\circ}(x^{i^\circ}, \vartheta^i) = e^{i^\circ} >=< E^{j^\circ}(x^{j^\circ}, \vartheta^j) = e^{j^\circ}$.

[18] We intend for voucher a given amount of money that can be used uniquely for buying the good the voucher has been issued for (in the present case the good *x*).

[19] This result sensibly differ from that obtained by H&R (2006). Because they assume identical utility functions and income, they can state that this policy is able to get a second best. In our scenario where utility functions may vary among households as well as income, this policy may result ineffective in reaching a second best optimum.

Assuming *j* the recipient and *i* the contributor, then a second best outcome is implementable only under the condition that $e^{i^\circ} \le e^{j^\circ}$.

proof

Let's define the new budget constraint for the recipient household:

$$y^j + E^j(\vartheta^j, x^j) = R^j + \tau 1 + \tau 2 \quad \text{with} \quad e^j \ge \tau 1$$

Where τ1 is the value of voucher that can be used exclusively to purchase good *x*. Voucher induce the household to spend at least $\tau 1$ on care, in order not to miss it out. The residual component τ2 represents the lump sum transfer. The condition $\tau 1 + \tau 2 = \tau$ has to be met, where τ is the sum of the voucher for *x (τ1)* and the lump sum *(τ2)* that the recipient household gets. Furthermore τ has to be set by the social planner in order to satisfy the contributor's participation constraint.

Let's assume that the second best optimal expenditure for the recipient is:

$E^{i^\circ}(x^{i^\circ}, \vartheta^i) = e^{i^\circ}$ then the second best optimal amount of good *x* is $x^{j^\circ} = F^j(e^{i^\circ}, \vartheta^j)$ and the second best optimal amount of good *y* (from the budget constraint) is: $y^{j^\circ} = R^j + \tau 1 + \tau 2 - e^{j^\circ}$.

Equivalently we can define the optimal values for the contributing household: $x^{i^\circ} = F^i(e^{i^\circ}, \vartheta^i)$ and $y^{i^\circ} = R^i + \tau - e^{i^\circ}$.

Thus the participation constraint for the taxed household is:

$$U^{i^\circ}[x^{i^\circ} = F^i(e^{i^\circ}, \vartheta^i), y^{i^\circ} = R^i + \tau - e^{i^\circ}] \ge U^{i^\circ}[x^{ic} = F^c(e^{j^\circ}, \vartheta^i, \vartheta^j), y^{ic} = R^i + \tau 1 + \tau 2 - e^{j^\circ}]$$

or equivalently

$$U^{i^\circ}[F^i(e^{i^\circ}, \vartheta^i), R^i + \tau - e^{i^\circ}] \ge U^{i^\circ}[F^c(e^{j^\circ}, \vartheta^i, \vartheta^j), R^i + \tau 1 + \tau 2 - e^{j^\circ}]$$

Where the *c* index indicates the values corresponding to the *cheating* strategy when a household type mimics the other type. Because we have assumed $e^{i°} \le e^{j°}$, then the contributing region, when mimicking the other type, can increase its level of care ($x^{ic} > x^{i°}$). What about the amount of good *y* the cheating household is able to purchase? Looking the contributor's budget constraint in the two scenarios (honest behaviour and cheating) and recalling that by hypothesis $\tau = \tau 1 + \tau 2$ and $e^{i°} \le e^{j°}$, it clearly emerges that the disposable income, after the expenditure on *x* is such that $y^{ic} < y^{i°}$.

When the contributing household chooses not to sincerely reveal his type then a gain in terms of good *x* is expected, but at the same time a loss in terms of good *y* is also expected. Because $x^{i°}$ and $y^{i°}$ are the values autonomously chosen by the household in a non distorted scenario $U_x^{i°} \big/ U_y^{i°} = E_x^{i°}$, then it is reasonable to expect in the cheating scenario the following inequality to hold $U_x^{ic} \big/ U_y^{ic} < E_x^{ic}$.

If the social planner sets the tax/subsidy mix in a second best scenario, then the contributor's participation constraint should be met and the cheating strategy should be avoided.

On the other hand the receiving household maximizes his utility function under the budget constraint which now is "constrained" by the voucher policy, i.e., $e^{j°} \ge \tau 1$.

It is straightforward to prove that the utility that the household gets from $e^{j\wedge} < \tau 1$ $(U^{j\wedge}[F^{j\wedge}(e^{j\wedge}, \vartheta^j), R^j + \tau 2 - e^{j\wedge}]$, where ^ indicates the values the household sets when the requirement to get the voucher *τ1* is not fulfilled) is lower with respect to the utility in the case that the expenditure is set in order to meet the constraint $e^j \ge \tau 1$ $(U^{j°}[F^j(e^{j°}, \vartheta^j), R^j + \tau 1 + \tau 2 - e^{j°}])$. In fact the household decision to set $e^j < \tau 1$ would determine a lower level both of care (good *x*) and of good *y*, and as a consequence a net utility loss.

Summing up it is possible to state that the social planner is able to attain a second best optimum when implementing this policy but only if the condition $e^{i°} \le e^{j°}$ is met.

5. Concluding remarks

The paper considers a social planner aiming at social welfare maximization when households privately provide the elderly care. Assuming that the social planner pursues his purpose by means of lump sum transfer, we characterize the welfare optimum both under full and asymmetric information. Under complete information it turns out that the lump sum transfer suitable to get the Pareto outcome, by which it is possible to implement the optimal level of elderly care, has to be set in such a way to induce equality in the marginal utility of the composite good *y* which is tantamount to say that equality in marginal utility of income has to be reached. From the analysis emerges that the best resource allocation might require a income transfer from the high severity / high cost type to the low severity / low cost type, where intuitively one would expect the opposite. The transfer sign depends on the sign of the elasticity of the marginal utility from care with respect to the cost elasticity of care. If the marginal caring effort increases much more steeply in severity than the benefit the family obtains from this, then it may be optimal for transfers to flow from the severe type to the less severe type.

In the context of information asymmetry, in which the social planner is unable to observe neither the level of care provided nor the household's type, we might expect a second best outcome. However, even a first best is still a possible equilibrium. If the recipient of the transfer has a higher gross income to begin with, a first best solution can be implemented. This occurs because, under the afore mentioned condition, the incentive compatible constraint, required to avoid a cheating strategy by the contributing household, does not bind and a first best level of elderly care is achievable. Relaxing this hypothesis on income, it turns out that the second best outcome requires, in order to be implemented, that the level of recipient's expenditure on care has to be forced upwards towards a certain target. This result derives from the fact that lump sum transfer in a second best scenario might avoid the receiving household to meet his Nash efficiency conditions. As a result we obtain distortions in the recipient's spending decision with reference to the elderly care good.

However, differently from the existing literature, we allow for an exception to the afore well-established rule. In fact in our very general setting no distortion occurs when the marginal effect on the contributor's utility of the recipient's expenditure on x is equal to the contributor's marginal utility with respect to the recipient's expenditure on good y. Furthermore it could also emerge the result that the social planner had to force the recipient to curb (i.e., to force downwards) the expenditure on elderly care, with respect to his attitude, in order to attain the second best outcome. This result, even if counterintuitive, follows from the risk of type misrepresentation that could be carried on by the taxed household in such a general setting.

Finally, starting from the consideration that lump sum transfer determines the mentioned distortion at the recipient, we investigate another policy at social planner disposal. It is shown that a voucher to be spent on good x along with a lump sum transfer might be able to reach a second best outcome, avoiding any distortion. This result holds under the condition that the expenditure on elderly care of the receiver is equal or greater to the expenditure on care faced by the contributor.

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Appendix 1

The quantity of good *(x)* is produced by households according to the production function:

$x = \xi(\ell, \vartheta)$

where $x_\ell > 0$, $x_{\ell\ell} < 0$; $x_\vartheta < 0$, $x_{\vartheta\vartheta} \geq 0$

Using the implicit function theorem we can write:

$\ell = \zeta(x, \vartheta)$; $\ell_x > 0$, $\ell_{xx} > 0$; $\ell_\vartheta > 0$, $\ell_{\vartheta\vartheta} \geq 0$

While the first and second derivatives of ℓ with respect to good x come as a direct consequence of the fact that $x_\ell > 0$, $x_{\ell\ell} < 0$; the sign of ℓ_ϑ can be derived in the following way:

$dx = \xi_\ell d\ell + \xi_\vartheta d\vartheta$ and $d\ell = \zeta_x dx + \zeta_\vartheta d\vartheta$

Substituting the former in the latter:

$d\ell = \zeta_x[\xi_\ell d\ell + \xi_\vartheta d\vartheta] + \zeta_\vartheta d\vartheta$

Dividing both sides by $d\vartheta$:

$$\frac{d\ell}{d\vartheta} = \zeta_x[\xi_\ell \frac{d\ell}{d\vartheta} + \xi_\vartheta] + \zeta_\vartheta$$

where $\zeta_x = \dfrac{\partial \ell}{\partial x}$ and $\xi_\ell = \dfrac{\partial x}{\partial \ell}$

Thus: $\zeta_\vartheta = -\zeta_x \xi_\vartheta$ or equivalently: $\dfrac{\partial \ell}{\partial \vartheta} = -\dfrac{\partial \ell}{\partial x}\dfrac{\partial x}{\partial \vartheta} > 0$

Appendix 2

Using (1), we know that $\dfrac{\partial U^i}{\partial \tau^i} = \dfrac{\partial U^i}{\partial x^{i*}}\dfrac{\partial x^{i*}}{\partial \tau^i} + \dfrac{\partial U^i}{\partial y^{i*}}\dfrac{\partial y^{i*}}{\partial \tau^i}$, and, using (2), we can write $\dfrac{\partial U^i}{\partial \tau^i} = U^i_y \left(E^i_x \dfrac{\partial x^{i*}}{\partial \tau^i} + \dfrac{\partial y^{i*}}{\partial \tau^i} \right)$. By differentiating the households' budget constraint

$R^i + \tau^i = y^{i*} + E^i(\vartheta^i, x^{i*})$ we get $1 = \frac{\partial y^i}{\partial \tau^i} + E_x^i \frac{\partial x^i}{\partial \tau^i}$ (it is simply obtained dividing by $d\tau$ the equation $d\tau^i = \frac{\partial y^i}{\partial \tau^i} d\tau^i + E_x^i \frac{\partial x^i}{\partial \tau^i} d\tau^i$). It follows that eq. (4) implies eq. (3), that is in the Pareto equilibrium, which is characterized by $U_y^{h*} = U_y^{l*}$, the optimum transfer outlays $\tau^h = -\tau^l$ induces equal marginal utilities for good y for all the households.

Appendix 3

The level of the subsidy/tax τ should be set as to equalize the marginal utility of consumption (y) across types $\vartheta \in \{\vartheta^l, \vartheta^h\}; \vartheta^l < \vartheta^h$. The counter-intuitive result where the high type is taxed and the low type is subsidized then requires $(U_y^h < U_y^l)\big|\tau = 0$. Assuming continuity, this condition is equivalent to

$$\frac{dU_y}{d\vartheta} = U_{yy}\frac{dy}{d\vartheta} + U_{yx}\frac{dx}{d\vartheta} < 0 \qquad (*)$$

By regularity, $U_{yy} < 0$; $U_{yx} \geq 0$. Here, x denotes the "benefit from care" which is attained when an amount $e=E(x,\vartheta)$ is expended (which, in our model, is equivalent to an opportunity cost of time). It is assumed that $E_x>0; E_{xx}\geq 0; E_\vartheta>0;\ E_{x\vartheta}>0$, i.e., higher levels of caring benefit require progressively) more inputs and more severe types require more inputs (at the margin) to attain any given benefit.

It can be checked (elsewhere in the paper) that $\frac{dx}{d\vartheta} < 0$. For τ=0, the household's budget constraint is given by $R=y+E(x,\vartheta)$ and by total differentiations we obtain for a given (potential) income R:

$$\frac{dy}{d\vartheta} = -\frac{dE}{d\vartheta} = -E_{\vartheta} - E_x \frac{dx}{d\vartheta}$$

Thus consumption y increases with severity if and only if the expenditure for long term care decreases. Note that technically this is possible if the higher cost of caring for more severe types is overcompensated by a reduction in the benefit of care that is generated.

Thus we obtain two possible cases:

i) expenditure is decreasing in severity, implying that $\frac{dy}{d\vartheta} > 0$. In this case the right hand side of (*) is unambiguously negative (even if $U_{yx}=0$);

ii) expenditure is increasing in severity, implying $\frac{dy}{d\vartheta} < 0$. In this case, the right hand side of (*) is positive only if $U_{yx}>0$.

Appendix 4

In consequence of the fact that the two households behave according to the Nash maximization rule, i.e., they move along the individual optimal path, the individual equilibrium point reflects the (first order) condition (see eq. 2) $U_y^{m*} = \frac{U_x^{m*}}{E_x^{m*}}$, $m \in \{l, h\}$, eq. 8 can then be rewritten as follows:

$$\Delta U_y^* = \frac{U_x^{i*}}{E_x^{i*}} - \frac{U_x^{j*}}{E_x^{j*}} = \frac{U_x^{i*} E_x^{j*} - U_x^{j*} E_x^{i*}}{E_x^{i*} E_x^{j*}} \qquad (**)$$

Hence the initial hypothesis $\Delta U_y^* > 0$ turns out to be met when the numerator of equation (**) is greater than zero ($U_x^{i*} E_x^{j*} - U_x^{j*} E_x^{i*} > 0$) given that the denominator is always positive.

Assuming:

$\Delta E_y^* = E_x^{i*} - E_x^{j*} >,< 0$ and $\Delta U_x^* = U_x^{i*} - U_x^{j*} >,< 0$, it follows that:

$\Delta U_y^* > 0$ if $\left(\Delta U_x^* + U_x^{j*}\right)E_x^{j*} - \left(\Delta E_x^* + E_x^{j*}\right)U_x^{j*} > 0 \quad \Rightarrow$

$\Delta U_x^* E_x^{j*} - \Delta E_x^* U_x^{j*} > 0 \Rightarrow \Delta U_x^* E_x^{j*} > \Delta E_x^* U_x^{j*}$ or $\dfrac{\Delta U_x^*}{U_x^{j*}} > \dfrac{\Delta E_x^*}{E_x^{j*}}$.

Appendix 5

$x_i^j = \psi_x^j(e^i, \vartheta^j)$ is derived as follows: $E^i(x^i, \vartheta^i) = e^i$ is the expenditure on care that the receiving household sets in order to maximize its utility. The cheating household *j* will set its expenditure on elderly care $E^j(x_i^j, \vartheta^j) = e^j$ so that its cheating cannot be established. This goal is attained when $E^j(x_i^j, \vartheta^j) = e^j = e^i$. Using the implicit function theorem and naming x_i^j the good provided by household *j* when pretending to be the other type: $x_i^j = \psi_x^j(e^i, \vartheta^j)$

Appendix 6

Social planner maximization problem with asymmetric information; first order conditions (focs):

$$\frac{\partial L}{\partial x^i} = U_{x^i}^i - \lambda^i E_{x^i}^i + \mu^i U_{x^i}^i - \mu^j U_{\psi^j}^j \psi_{e^i}^j E_{x^i}^i \le 0, \quad x^i \ge 0, \quad x^i\left(\frac{\partial L}{\partial x^i}\right) = 0 \quad \text{(a.1)}$$

$$\frac{\partial L}{\partial x^j} = U_{x^j}^j - \lambda^j E_{x^j}^j - \mu^i U_{\psi^i}^i \psi_{e^j}^i E_{x^j}^j + \mu^j U_{x^j}^j \le 0, \quad x^j \ge 0, \quad x^j\left(\frac{\partial L}{\partial x^j}\right) = 0 \quad \text{(a.2)}$$

$$\frac{\partial L}{\partial y^i} = U_{y^i}^i - \lambda^i + \mu^i U_{y^i}^i - \mu^j U_{y^i}^j \le 0, \quad y^i \ge 0, \quad y^i\left(\frac{\partial L}{\partial y^i}\right) = 0 \quad \text{(a.3)}$$

$$\frac{\partial L}{\partial y^j} = U_{y^j}^j - \lambda^j - \mu^i U_{y^j}^i + \mu^j U_{y^j}^j \le 0, \quad y^j \ge 0, \quad y^j\left(\frac{\partial L}{\partial y^j}\right) = 0 \quad \text{(a.4)}$$

$$\frac{\partial L}{\partial \tau} = \lambda^i - \lambda^j = 0 \quad \text{(a.5)}$$

$$\frac{\partial L}{\partial \lambda^i} = y^i + E^i(\vartheta^i, x^i) - R^i - \tau = 0 \qquad (a.6)$$

$$\frac{\partial L}{\partial \lambda^j} = y^j + E^j(\vartheta^j, x^j) - R^j + \tau = 0 \qquad (a.7)$$

$$\frac{\partial L}{\partial \mu^i} = U^i[x^i, y^i] - U^i\left\{\psi_x^i[E^j(x^j, \vartheta^j), \vartheta^i], y^j\right\} \geq 0, \quad \mu^i \geq 0,$$

$$\mu^i(\frac{\partial L}{\partial \mu^h}) = 0 \qquad (a.8)$$

$$\frac{\partial L}{\partial \mu^j} = U^j[x^j, y^j] - U^j\left\{\psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j], y^i\right\} \geq 0, \mu^j \geq 0, \mu^j(\frac{\partial L}{\partial \mu^j}) = 0 \qquad (a.9)$$

Analysing eqs. a.2 and a.4 we note that, with reference to the j severity type which is assumed to be the contributing one, the condition $SMS_{x,y}^j = E_x^j$ has still to be met. This condition coincides with that identified for efficiency in the scenario of complete information.

Eq.a.5 suggests how the transfer τ has to be set by the social planner in order to equalize the shadow price of income of the two households, or in other words to get: $\lambda^i = \lambda^j$, but this latter in turn implies that the difference in marginal terms between the two type of households with respect to the composite good cannot be settled up.

Indeed, from a.3 and a.4 clearly emerges that: $U_y^i > U_y^j$, i.e., the incentive compatible constraint does not permit to join the condition of equality between the marginal utility (with respect to the compos good) for the two household types.

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